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تصميم ترميزات الـ LDPC بمصفوفة اختبار متكافئ بمواصفات جيدة

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الملخص:

تعد ترميزات الاختبار المتكافئ ذات الكثافة المنخفضة (LDPC) واحدة من الترميزات المهمة وذلك لخواصها الجيدة، تستخدم ترميزات LDPC بشكل واسع في العديد من التطبيقات، نذكر منها على سبيل المثال لا الحصر: أنظمة الاتصالات المتنقلة من الجيل الرابع والخامس، اتصالات الإنترنت اللاسلكية، الراديو الرقمي، والتلفازالرقمي عبر القمر الصناعي أو الكابل. وهذا ما جعل العديد من الأبحاث تهتم بهذه الترميزات.

سنقدم في هذه المقالة خوارزمية فعالة تقوم بتصميم مصفوفات الاختبار المتكافئ لترميزات LDPC. هذه الخوارزمية توازن بين المسافة الصغرى للترميز والتناثر الخاص بمصفوفته، نستنتج منها مباشرة المصفوفة المولدة مما يسمح بتوليد كلمات الترميز بسهولة دون الحاجة لحل جملة معادلات، كما أن هذه المصفوفة ذات رتبة سطرية كاملة.

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Design LDPC Codes with Good Characteristic Parity Check Matrix

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Abstract

Considered essential for their favorable characteristics, Low-Density Parity-Check codes (LDPC codes) are widely used in various applications such as fourth and fifth-generation mobile communication systems, wireless internet connections, digital radio, and digital TV via satellite or cable. This widespread usage has prompted numerous research efforts dedicated to addressing LDPC code-related issues.

In this paper, we introduce an effective algorithm to design parity check matrices for LDPC codes. This algorithm balances between the code's minimum distance and the sparsity of its matrix. Moreover, the generator matrix is produced directly from the parity check matrix, making encoding straightforward. Hence, there is no need for solving equations. Furthermore, these matrices are full rank

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1. Introduction

LDPC codes stand out for their efficiency, enabling practical implementation that nearly reaches the Shannon channel capacity for reliable transmission. Furthermore, these codes come with the benefits of simplified algorithms, providing greater speed and accuracy; see (Onverwagt, 2023). That makes them widely used in many applications likes: fourth generation (4G) and fifth generation (5G) mobile communication systems, wireless internet connections, digital radio and digital TV via satellite or cable; (Rana, 2019).

Developed in the early 1960s, LDPC codes have seen a remarkable resurgence in recent years, becoming widely embraced by modern communication standards. Achieving high processing speed and energy efficiency requires maintaining a low level of complexity in both the encoding and decoding processes; see (Onverwagt, 2023).

Defined by parity check matrices, LDPC codes enable efficient decoding. However, encoding with low complexity proves challenging due to the generally unknown generator matrix; see (Qi et al, 2013). Consequently, LDPC codes demand an effective strategy for encoding.

In this paper, we will present an algorithm used to design parity check matrices for LDPC codes. This matrix H has the following properties: firstly, we can easily compute the generator matrix G of the LDPC code, making the encoding of this code straightforward. In LDPC code, the encoding is typically achieved by solving equations $Hc^T = 0$; (Onverwagt H., 2023, page 14), but here we find the codeword c by the product $c = aG$, where a is the message belonging to $\{0,1\}^k$. Secondly, this algorithm provides a LDPC code with a good minimum distance. Hence, we strike a balance between this distance and the sparsity of the parity check matrix. Additionally, the provided parity check matrix has full row.

2. Literature Review

LDPC codes are very popular, prompting many researchers to address the design and encoding of these codes. They are striving to develop faster techniques capable of encoding and decoding these codes.

In the paper by (Qi et al., 2013), the encoding of LDPC codes is based on the known concept of approximate lower triangulation. The paper presents the greedy permutation algorithm to transform parity check matrices into an approximate lower triangular matrix. The paper by (Prasartkaew et al, 2013) introduces a construction algorithm for short block irregular LDPC codes, applying a magic square theorem as a part of the matrix construction. In (Sharifi et al, 2015), they design a code for two-user Gaussian multiple access channels. In (Broulim , 2018) two main methods are proposed, a method based on backtracking codeword estimations and a method based on using several parity check matrices. The second method, so called Mutational LDPC, utilizes several parity check matrices produced by slight mutations which run in parallel decoders. A low-complexity LDPC encoder for space applications is implemented in the study by (Wang et al, 2021). RAM is used to cache the vector for multiplication of the sparse matrix and vector, and the write address and read address of the RAM are generated by two counters, consuming significantly fewer hardware resources. Meanwhile, the SRAA circuit is exploited to implement the multiplication of the dense core matrix and vector. The paper by (Boiko et al, 2021) developes models for efficient coding in information networks based on codes with a low density of parity check. As, number of decoding iterations is taken for envisaging the defined noise immunity indices. The paper by (Ebert et al, 2023) presents sparse regression LDPC codes and their decoding. A sparse regression code is a concatenated structure with an inner SPARC-like code and an outer non-binary LDPC code. The inner code is decoded using AMP with a dynamic denser, which runs BP on the factor graph of the outer LDPC code to improve the state estimate at every iteration.

The most of the encoding works are done in solving the system $Hc^T = 0$ effectively, where H is the parity check matrix. Providing LU and other factorization to H will give less sparse matrix. In this paper we work by providing the matrix H ,which gives a straightforward generator matrix G. Then we encode by using the multiplication $c = aG$ instead.

3. LDPC Codes:

The LDPC codes are defined in terms of sparse parity check matrices over the field F_2. Suppose a parity check matrix H associated with the linear block code C. If the matrix H is sparse, the code C is said to be the LDPC code. The matrix is considered to be sparse if it has a small number of nonzero-elements in each row and column.

If the columns weight in the matrix H are all the same and rows weight are all the same, the LDPC code is called regular. If not, it is irregular. It has been shown that irregular codes perform better; see (Broulim J., 2018, page 25).

Let the original message be $a \in F_2^{\ k}$. This message is encoded by the code C with the codeword c. That is given by $c = aG$, where $c \in F_2^{\ n}$. The generator matrix G has the dimensions $k \times n$, while its parity check matrix H has the dimensions $(n-k)\times n$; see (Guruswami, 2010).

The LDPC code usually is defined in terms of its parity check matrix H. So, in the next section, we are going to provide an algorithm which is used to design a sparse parity check matrix for LDPC codes, namely; the SPCM algorithm. The produced matrix has the standard form $H = [P \quad I]$. Then, the code's generator matrix G will be straightforward. It will be given by $G = [I \quad P^T]$.

4. Designing Parity Check Matrix:

In the following algorithm, we are going to design parity check matrix H , which has the dimensions $n \times m$, where $m = n - k$. The matrix $H = [P \quad I]$. The SPCM (sparse parity check matrix) algorithm is designed the matrix P , which has the dimension $m \times (n-m)$. Let P_i be the i^{th} column of the matrix P.

4. 1. The SPCM Algorithm:

Let $s = n - m, f = 0, c = m$

For $i = 0, \ldots, \left\lfloor \frac{s-1}{2} \right\rfloor$

If $\left(\left\lfloor \frac{c}{2} \right\rfloor \geq 1 \right)$ then

If (c is even) then

Let $c = \left\lfloor \frac{c}{2} \right\rfloor$, $f = 2i+1$

$P_{2i+1} =$ $((\underbrace{1....1}_{c} \; \underbrace{0....0}_{c}$: repeat to reach m index)

Let P_{2i+2} be the flipping indexes of P_{2i+1}, (where 0 become 1 and vice versa).

Else

Let $c = \left\lfloor \frac{c+1}{2} \right\rfloor$, $f = 2i+1$

$P_{2i+1} = ((\underbrace{1....1}_{c} \; \underbrace{0....0}_{c-1} \; \underbrace{1....1}_{c-1} \; \underbrace{0....0}_{c}):$

: repeat to reach m index)

Let P_{2i+2} be the flipping indexes of P_{2i+1}, (where 0 become 1 and vice versa).

Stop when $(f = s)$ or $\left(\left\lfloor \frac{c}{2} \right\rfloor \le 1 \right)$.

4. 2. The extension of SPCM Algorithm:

Usually n is much greater than n-m. However, if the SPCM algorithm does not generate all the columns of P, we have generated the remaining columns of P from the odd columns of P as the following:

$$\{P_1+P_3, P_1+P_5, ..., P_1+P_f, P_3+P_5, ..., P_5+P_f, ..., P_{f-2}+P_f, P_1+P_5+P_7, ...,$$
$$P_{f-4}+P_{f-2}+P_f, ..., P_1+P_3+...+P_f\}.$$

Example:

Let $n=12, m=7$ the SPCM algorithm gives the matrix P :

$$P = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}, \text{That gives } H$$

$$H=\begin{bmatrix}1&0&1&0&1&1&0&0&0&0&0&0\\1&0&1&0&0&0&1&0&0&0&0&0\\1&0&0&1&1&0&0&1&0&0&0&0\\1&0&0&1&0&0&0&0&1&0&0&0\\0&1&1&0&1&0&0&0&0&1&0&0\\0&1&1&0&0&0&0&0&0&0&1&0\\0&1&0&1&1&0&0&0&0&0&0&1\end{bmatrix}$$

Its generator matrix G will be given by

$$G=\begin{bmatrix}1&0&0&0&0&1&1&1&1&0&0&0\\0&1&0&0&0&0&0&0&0&1&1&1\\0&0&1&0&0&1&1&0&0&1&1&0\\0&0&0&1&0&0&0&1&1&0&0&1\\0&0&0&0&1&1&0&1&0&1&0&1\end{bmatrix}$$

If the original message is $a=11001$, then the codeword c will be

$c=aG=110010101010$.

Example:

Let $n=20, m=10$, The SPCM algorithm and its extension gives the matrix P :

$$P=\begin{bmatrix}1&0&1&0&1&0&1&0&0&0\\1&0&1&0&1&0&0&1&0&0\\1&0&1&0&0&1&1&0&0&1\\1&0&0&1&1&0&0&1&1&0\\1&0&0&1&0&1&1&0&1&1\\0&1&1&0&0&1&0&1&1&0\\0&1&1&0&1&0&1&0&1&1\\0&1&0&1&1&0&0&1&0&1\\0&1&0&1&0&1&1&0&0&0\\0&1&0&1&1&0&0&1&0&1\end{bmatrix}$$, That gives $H=[P\,|\,I]$

Its generator matrix G will be given by

$$G=\begin{bmatrix}1&0&0&0&0&0&0&0&0&0&1&1&1&1&1&0&0&0&0&0\\0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&1\\0&0&1&0&0&0&0&0&0&0&1&1&1&0&0&1&1&0&0&0\\0&0&0&1&0&0&0&0&0&0&0&0&0&1&1&0&0&1&1&1\\0&0&0&0&1&0&0&0&0&0&1&1&0&1&0&0&1&1&0&1\\0&0&0&0&0&1&0&0&0&0&0&0&1&0&1&1&0&0&1&0\\0&0&0&0&0&0&1&0&0&0&1&0&1&0&1&0&1&0&1&0\\0&0&0&0&0&0&0&1&0&0&0&1&0&1&0&1&0&1&0&1\\0&0&0&0&0&0&0&0&1&0&0&0&0&1&1&1&1&0&0&0\\0&0&0&0&0&0&0&0&0&1&0&0&1&0&1&0&1&1&0&1\end{bmatrix}$$

If the original message is $a=0110001100$, then the codeword c will be

$c=aG=01100011000001111000$.

5. Results and Discussion:

We provide the SPCM Algorithm and its extension to design a parity check matrix for the LDPC codes. These matrices are in standard form, so the generator matrix of the LDPC code is computed directly. That makes the encoding very easy and straightforward. These algorithms provide a code with good minimum distance, as we balance between this distance and the sparsity of the parity check matrix. As, we design the matrix P with reasonable number of 1's in each row, which are located to make the minimum distance as large as possible. In addition, the parity check matrix has full row rank, since the standard form guarantees that.

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